Simple Dynamical Systems Modeling

Change as Outcome Models

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Emergence

“Let’s consider emergence in a fully defined arena, like checkers or chess. The rules of the game, though few in number, give rise to a huge number of possibilities, most of them irrelevant or outright bad if the objective is to win the game. However, there are certain recurring patterns (regularities) among these possibilities that greatly influence the possibility of winning. The regularities even have names, like “sacrifice,” “pin,” and “gambit.” These regularities depend upon particular interactions between the pieces. The regularities obey “rules” at a higher level. These new rules (strategies) for exploiting the regularities cannot contradict the rules of the game, but they are emergent in the sense that they are not obtainable by simply “summing up” the rules applying to individual pieces.” – John Holland

Dynamical Systems Theory is Based on Emergence

How multi-component systems generate patterns that cannot easily be reduced to its parts

I like to say we are Quantifying the Qualitative

• We have Qualitative Descriptors for Psychological Phenomena
• Systems assumes that what we are observing are differences in the temporal characteristics
• Requires a strong understanding of temporal patterning
  – It is then possible to also get some semblance of emergence too
Three Parts of Variability

Temporal Patternings Type I
- Constant Value – The value is unchanging through time
- Linear Change – The value is changing at a constant rate through time (constant velocity)
- Simple Nonlinear Change – The value is changing at a nonconstant rate through time (velocity is changing)

Temporal Patternings Type 2
- Constant Oscillations – The Oscillatory Qualities are Constant
- Linear Changing Oscillations – The Oscillatory qualities is changing at a constant rate through time
- Nonlinear Change in Oscillations – The Oscillatory Qualities are changing at a nonconstant rate through time

Temporal Patternings Type 3
- Deterministic Chaos – Irregular to the point of having properties akin to randomness, but actually deterministic
- Chaos is known for its sensitivity to initial conditions and its exponential cost in prediction
- Sensitivity to Initial Conditions – Where it starts dramatically changes the temporal pattern
- Exponential Cost in Prediction – Predicting slightly further into the future requires exponentially more data
The Temporal Signature is a Combination of the Types

- Example Combo of Type 1 linear change and Type 2 constant oscillations (Did my best)

Distinguishing Error and Perturbations is a Little Trickier

- The Temporal Pattern as a Function of Emergence
- Perturbations as a Function of Open Systems (more to the system than what we are examining)
- Measurement Error (Imprecision)

Let’s Start with a Constant Value

- We Never See This
- Here Is More Like it

Notice that it is difficult to Differentiate the Three Parts and the Parts from the temporal pattern (but we need to)

Example 1: Measurement Error

- To Represent Measurement Error, I am going to take my constant value at each point in time and add a random number
- $10 + \text{Random Number} \sim \text{Normal distribution with mean} = 0 \text{ and } \text{sd} = 1$

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Constant

Imprecision of Measurement
Example 2: Nonstationarity

- Now, I am adding the same random number, but the logic is that this new value each time is the new constant value
  - I am feeding the numbers into the equation of the constant value

```
Nonstationary
```

Drift in the Constant Value

Example 3: Perturbations

- Adding the same random number, but the logic is that the temporal pattern is re-establishing itself
  - I am feeding the numbers into the equation of the constant value, but the constant value has stable properties and thus countering drift

```
Stability
```

Constantly Being Perturbed

Three Examples are Similar/Same in Time Series, Not in Change

```
Constant

Nonstationary

Stability
```

The Relationships Between Value and Change are Different!!

Dynamical Systems Assumes Stability of Temporal Patternings

- Systems are constantly being perturbed
  - However, our observance of the temporal patterns is despite the constant perturbations
- This is the notion of stability
- Stability comes out in the relationships between derivatives
  - Like Change’s relationship to value
    • First derivative to the zero derivative
  - So do the different kinds of temporal patternings
Change as Outcome

• Some outcome (X) measured through time
• Plot Change in X (difference between future and current X) on the Y axis
  — Value (X) is on the X axis
• Add the Best Fitting Straight Line

Let’s say the Slope is Negative

What Happens When X is above the Set Point

• When the Slope of the Line is Negative AND the Value of X is above the Set Point
  — We Observe Negative Change

In the Next Moment in Time the Value moves towards the Set Point

Change as Outcome

• Where Change = 0 is particularly informative
  — Known as a set point
  — Example, Self Efficacy you move towards or away from

Point at which there is no change

What Happens When X is below the Set Point

• When the Slope of the Line is Negative AND the Value of X is below the Set Point
  — We Observe Positive Change

In the Next Moment in Time the Value moves towards the Set Point
Negative Slopes Indicate Attraction to the Set Point

- Notice how this inherently captures the idea of resistance to perturbations
  - Get knocked away from the set point and you move back towards it

\[ (X_{t+1} - X_t) = b_0 + b_1(X_t) + e_t \]

- We have a simple regression equation where value predicts change in value
  - When \( b_1 \) is negative, it seems to indicate attraction
- The Regression is for one time series (through time)
  - Or multiple Pre Post (two points in time)
  - Otherwise we violate dependency assumptions

\begin{align*}
\text{Let's Do a Little Exploring of this Equation}
\end{align*}

Positive Slopes Still have a Set Point

- We already know when \( b_1 \) is negative it shows attractive behavior
  - The set point is when change is zero
    - \(-b_0/b_1\)
  - What does the a positive slope do?

\begin{align*}
\text{Value of } X \text{ Where it stays there}
\end{align*}

However, what happens around the set point is very different
What Happens When X is above the Set Point

- When the Slope of the Line is Positive AND the Value of X is above the Set Point
  - We Observe Positive Change

In the Next Moment in Time the Value moves away from the Set Point

What Happens When X is below the Set Point

- When the Slope of the Line is Positive AND the Value of X is below the Set Point
  - We Observe Negative Change

In the Next Moment in Time the Value moves away from the Set Point

Positive Slopes Indicate Repulsion from the Set Point

- So the sign of the slope when negative indicates attraction
- When positive it indicates repulsion

What does the steepness of the Slope Represent?

Steepness of the Slope Indicates the Strength of Attraction/Repulsion

The Shallower Slope on the Right moves cases towards the set point more slowly than the one of the Left
Summary So Far

• When Change is the Outcome the Slope indicates attraction/repulsion
• The Sign tells us which is occurring
  – Negative is Attraction
  – Positive is Repulsion
• Both have set points!!
  – Attraction is stable because if you move off, you try to move back
  – Repulsion is unstable because if you move off the set point you continue to move away
• The steepness of the slope is the strength of attraction/repulsion
  – Steeper slope, stronger attraction/repulsion
  – In fact, they can be so strong as to force you to overshoot it!!

Let’s Do a Little More Exploration

\[ (X_{t+1} - X_t) = b_0 + b_1(X_t) + e_t \]

• When \( b_1 \) is zero the Intercept (\( b_0 \)) takes on the average change
  – This can be no change (\( b_0 = 0 \))
  – This can be constant change (\( b_0 = \) constant rate of change)
• So, this approach defaults to the equivalent of constant or no growth

In Fact, All Type 1 Change can be Captured by this Equation

First Order Change Captures All Forms of Type 1 Change

Constant Value
(At the Set Point or When \( b_0 = 0 \) and No \( b_1 \))
First Order Change Captures All Forms of Type 1 Change

\[(X_{t+1} - X_t) = b_0 + b_1(X_t) + e_t\]

- The Equation Above Unites the Notion of Perturbations and the Notion of All Type 1 Change together
  - It characterizes multiple patterns of change in terms of behavior around a set point
  - The slope indicates attraction vs. repulsion to the Set Point
  - The steepness of the slope indicates how attractive/repulsive the set point is

Real Data Example

- Daily Diary of Self Reported Anxiety for One teenager with Type 1 Diabetes
  
  - Three Parts
    - Temporal Pattern
    - Perturbations
    - Measurement Error
  
  - Believe it is a Type 1 temporal pattern (does not look oscillatory)

Making a Difference Score

*Select just this case (dyadid = 3). recode dyadid (3=1) (else=0) into keep. filter keep.

*Generate a Lead Variable One time. CREATE /tdanx2_1=LEAD(tdanx2 1). execute.

*Create a difference between Lead and Current compute t danx2_cha = t danx2_1 - t danx2. execute.
Plot of Change and Value

- Overlaid Best Fitting Straight Line
  - (Note the Effect Size)
  - Set Point is about 2.2
  - Attractive (Negative Slope)

Regression

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<th>Model</th>
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<th>Standardized Coefficients</th>
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<td>-.950</td>
<td>.289</td>
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</table>

- Set point at No Change
  - Pred Cha=2.044-.950(Anx)
  - 0=2.044-.95(Anx)
  - Anx = 2.15
- Slope Shows Activity Relative to Set Point
  - Negative (attractive)
  - Strength of attraction -.95

Expanding to a Cubic Equation

- The above graph implies an equation with three set points
  - Two with negative slopes are attractors
  - One with a positive slope is a repeller
- $(X_{t+1} - X_t) = b_0 + b_1 (X_t) + b_2 (X_t)^2 + b_3 (X_t)^3 + e_t$

Expanding to a Cubic Equation

- This implies two different possible behaviors where you are being drawn towards different set points as a function of where you are on X
  - We can imagine a big perturbation could push you across the repeller and now you would be under the influence of the other attractor
  - Two Basins of Attraction
Adding in Another Variable

\[ (X_{t+1} - X_t) = b_0 + b_1 X_t + b_2 Z_t + e_t \]

- \( Z \) shifts the entire line
  - It changes the location of the set point
  - Effects nothing else

Interacting with Another Variable

\[ (X_{t+1} - X_t) = b_0 + b_1 X_t + b_2 Z_t + b_3 XZ_t + e_t \]

- Allowing \( Z \) to interact with \( X \) alters the slope
  - This can strengthen or weaken attraction
  - Turn attraction into repulsion (flip the sign)
- Interactions are much more meaningful than main effects!!

We Can Even Combine The Two

- Above is the result of of the cusp catastrophe model which has two external variables moderating the system
  - When \( c = 0.5 \), one basin of attraction
  - When \( c = 1 \), two basins of attraction

Summary

- Expanding the relationship between \( X \) and Change in \( X \) allows for multistability
  - Multiple behaviors that differ by where you are in \( X \)
  - Perturbations move between them
- Adding in other variables alter the relationship between \( X \) and change in \( X \)
  - Can just shift the set point location
  - Can also (interaction example) alter the strength of attraction/repulsion and even change the kind of behavior
Beyond One Time Series

• The problem is that we have multiple Time Series
  — E.g. Daily Diaries
• Instead, we can use the identical equations in multilevel modeling
  — Build Change in X within person over time
  — Predict Change in X as a function of X over time using an expected relationship (e.g. linear, cubic)
  — Add in other predictors at level 1 and level 2 building based on what they should do (e.g. main effects, interactions)

Multilevel Code in SPSS

MIXED tdanx2_cha WITH tdanx2
  /FIXED=tdanx2 | SSTYPE(3)
  /METHOD=REML
  /PRINT=SOLUTION TESTCOV
  /RANDOM=INTERCEPT tdanx2 | SUBJECT(dyadid) COVTYPE(VC).

• Linear Model, allow for a random effect on intercept and slope
  — Different slopes and intercepts for each teenager

Multilevel Modeling Example

• Back to our Anxiety Example but now with Multiple Time Series
• Make Everyone’s Change Variable (but keep them separate)
  DATASET ACTIVATE DataSet1.
  SORT CASES BY dyadid.
  SPLIT FILE BY dyadid.
  CREATE /tdanx2_1=LEAD(tdanx2). execute.
  compute tdanx2_cha = tdanx2_1 - tdanx2. execute.
  split file off.

Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>df</th>
<th>t</th>
<th>Sig.</th>
<th>95% Confidence Interval</th>
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a. Dependent Variable: tdanx2_cha.

• Average Set Point (set change to zero, solve for anxiety) = 1.53
  — A little lower than my example case
• Average Slope = .81
  — A little weaker than my example case

Estimates of Covariance Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
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a. Dependent Variable: tdanx2_cha.

• Variance Components on Everything!
  — The location and strength of attraction vary by person
  — However, everyone is showing attractive behavior because zero if over 5 standard deviations of the variance component on the slope (.81/0.0199)
Let’s Add another Variable

- ADRR is average risk in blood glucose (scaled zero to 100 where 100 is max risk)
  - Adding it as a main effect and interaction
- Since it is a time varying covariate (varies over days) I am also adding random effects

MIXED tdanx2_cha WITH tdanx2 aadr
/FIXED=tdanx2 aadr aadr*tdanx2 | SSTYPE(3)
/METHOD=REML
/PRINT=SOLUTION TESTCOV
/RANDOM=INTERCEPT tdanx2 aadr aadr*tdanx2 | SUBJECT(dyadid) COVTYPE(VC).

Results

- When there is no Risk (ADRR=0)
  - Set Point is 1.52
  - Slope is -.76
- But ADRR alters the slope
  - For every one unit change in ADRR, the slope becomes more negative by .003
  - The attractive behavior becomes STRONGER!!

There are More Differences Across Individuals Remaining

- We see a VC on the anxiety by ADRR interaction
  - This could be because I did not apply any special centering
  - Group Centering would focus on what we call intrinsic dynamics (dynamics specific to the individual)
Two Simultaneous Equations

- Modeling two simultaneous equations allows for the notion of coupling and coordination
  - Coordination is about how two or more variables move together in time
- \( (X_{t+1} - X_t) = b_0 + b_1(X_t) + b_2(Y_t) + e \)
- \( (Y_{t+1} - Y_t) = b_3 + b_4(Y_t) + b_5(X_t) + e \)
  - \( b_2 \) and \( b_5 \) are known as coupling terms
  - They link the two equations to each other so that one is constantly perturbing the other

State Space

- The equivalent way to draw graphs of change is to make a plot of \( Y \) and \( X \) using arrows (vectors) to show the changes – where they are going in time

Type 1 and 2 Patterns of Change as a Function of two Linear Equations

- The Color, in this case, Represent the Relative Rates of Change

Summary

- Two Simultaneous Equations are capable of capturing all type 1 and type 2 patterns
- We now have two dimensions of change
  - The equations are linked through the coupling
- As with the one equation circumstance, linear equations are only capable of describing a single set point and its general behavior
- Adding in other variables, main effects just move the set points, interactions can alter the behaviors and their strengths
Actor Partner Models as a Way to Model Two Equations

- Create a new change variable called Z
  - Z is sometimes the Change in X
  - Z is sometimes the Change in Y
  - Add dummy codes that tell us when Z is X vs. Y
- \[ Z = b_1(X_{ind}) + b_2(X^*X_{ind}) + b_3(Y^*X_{ind}) + b_4(Y_{ind}) + b_5(X^*Y_{ind}) + e \]
  - Where e allows for different variances as a function of X_{ind} or Y_{ind}
  - Note X_{ind} = 1-Y_{ind}
  - This is a case of overdummy coding and hence has no intercept (b_1 and b_2 are the intercepts)

Example with 2 Simultaneous Equations

- Let’s rethink Anxiety and Diabetes Risk
  - Maybe they are both changing in time
  - That is, instead of ADRR moderating Anxiety, maybe they push and pull on one another
- One Dimension is Anxiety
- One Dimension is ADRR
- Make Change Variables for Each
  - Allow coupled equations

Syntax

- I already have change in Anxiety, just need ADRR

  ```
  DATASET ACTIVATE DataSet1.
  SORT CASES BY dyadid.
  SPLIT FILE BY dyadid.
  CREATE /adrr_1=LEAD(adrr 1).
  execute.
  compute adrr_cha = adrr_1 - adrr.
  execute.
  
  Syntax - Restructure
  
  - Restructure Datafile so that Change is sometimes Anxiety and sometimes ADRR

  *Need an extra of each predictor.
  compute adrr_2 = adrr.
  compute tdanx2_2 = tdanx2.
  execute.

  *Restructure to make change, own and coupling variables.
  VARSTOCASES
  /MAKE change FROM tdanx2_cha adrr_cha
  /MAKE own FROM tdanx2 adrr
  /MAKE coupling FROM adrr_2 tdanx2_2
  /INDEX=index1(2)
  /KEEP=dyadid entry2
  /NULL=KEEP.

  *Create dummy codes to indicate which variables working with.
  recode index1 (1=1) (2=0) into anx.
  recode index1 (1=0) (2=1) into adrr.
  execute.
  ```
**Multilevel Model**

MIXED change WITH own coupling adrr anx
/FIXED=own*anx coupling*anx adrr coupling*adrr | NOINT SSTYPE(3)
/METHOD=REML
/PRINT=SOLUTION TESTCOV
/REPEATED=anx | SUBJECT(dyadid) COVTYPE(DIG)

- Two simultaneous equations
  - Everything with the anx in it is when change in anxiety is the DV
  - Everything with the adrr in it is when adrr is the DV
- Allowed for different error variances

**Random Effects**

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- Variance Component on Coupling with Change in Anxiety as the DV
  - Some Folks have positive coefficient of ADRR on change in Anxiety
  - Some Folks have negative coefficient of ADRR on change in Anxiety
- The Average is no coupling relationship

**Fixed Effects**

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- On average, no coupling
  - Could change if I group centered or theorized interactions, etc.

**I Could Theorize Different Relationships**

- Having more than one stable pattern requires nonlinear equation forms (like the cubic one we examined before)
- Can add in variables that moderate various relationships
  - E.g. what might we expect to moderate the coupling of Risk in predicting change in Anxiety?
- Can control the existence of Random Effects to specify where we might and might not expect key moderators
- To Draw them in a State Space, we need a program other than SPSS (I like Grapher on the Mac, though we have code in R on my website)
1. Some Remaining Issues

- Pre Predicting Post Instead
  - I presented this all in terms of Change as Outcome, but Lead/Lag relationships Work Too (they are just not as intuitive)
- Takens’ Theorem
  - In a Single Outcome are All the Dynamics of All the Related Equations
- Other ways to get Type 2 Patterns
  - Moving to 2nd Order Change Models
- Type 3 Patterns and Beyond
- Timing Matters
  - Not Measuring Frequently Enough Can Miss the Pattern of Change
  - Measuring Too Frequently Adds Autocorrelations that have nothing to do with Change

2. Pre-Predicting Post

- In Pre Predicting Post (Right) a 45 degree line represents no change
  - The line is shifted by a constant

- So, can get same thing by using a lead/lag relationship
  - Notice difference in $R^2$ – statistically not identical (since we test against zero)
Takens’ Theorem
• It ends up that all the information of multiple variables can actually be found in the relationship amongst derivatives of a single variable

Three Coupled Equations Same as Three Lead/Lag Relationships
• Notice that I no longer need X, Y, and Z when we think about relationships
  – It is all in X!!

2nd Way to Get Type 2 Change
• Based on Takens’ Theorem there is thus a second way to get Oscillations
  – Option 1: Two simultaneous equations of change
  – Option 2: One equation of Change where the equivalent of Acceleration is the DV instead of Oscillations
• Acceleration is change in change
  – 2nd order derivative
• Change in Change in X=  b₀ + b₁X + b₂(Change in X) + e
  – This is also known as the equation for a damped oscillator
  – B₁ is a form of frequency, b₂ is a form of damping

2nd Order Equations Imply Two First Order Equations
• Both generate oscillations
• Two first order equations essentially argue that oscillations are the byproduct of the coupled relationships
• One 2nd order equation essentially argues that oscillations are occurring though less is known as to why
  – Sometimes folks couple two 2nd order equations, this implies 4 1st order equations
Type 3 Patterns of Change

• Three First Order Coupled Equations are capable of
  – All type 1 patterns
  – All type 2 patterns
  – All type 3 patterns (requires complex forms with interactions and nonlinear terms)
• So can two coupled acceleration as outcome models
  – These are also capable of type 4 patterns though there are no known stable unique patterns that are not already depicted
• The number of 1st order equations needed to depict a system is a form of the system’s dimensionality
  – We include those implied by moderators as dimensions

Timing Matters: Logic

• There is a timing under which the change of the temporal pattern is occurring
• Example: Takes 4 days to cycle and you measure it every 4 days
  – Looks like it never changes
• We need to oversample the pattern at which the change in the temporal pattern occurs to properly capture that temporal pattern
  – We need to measure frequently enough to see that it cycles over the four days

Last Lesson: Timing Matters

• Different Time Delays alter the ability to recapture the original relationships using Takens’ Theorem

Timing Matters: Logic

• However, if we measure every minute it is going to take many many measurements before we will start to observe cycles
  – Oversample too much and we generate a large temporal relationship that has nothing to do with the temporal pattern
  – The result is a distortion of the temporal pattern
• So, we want to oversample, but not by too much (just enough)
  – The more complicated the temporal pattern, the more oversampling we need
  – E.g. it only takes one measurement to show the constant value, two for a linear slope, and so forth
  – Timing is more important for Type 2 than type 1, and more important for Type 3 than type 2